

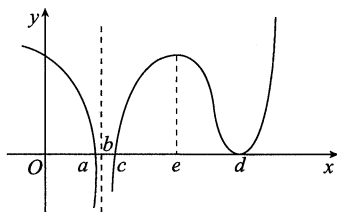
2016年

数学(三) 参考答案

一、选择题

(1) B

解 由导数 $f'(x)$ 的图形(见右图)可知 $f(x)$ 的驻点为 $x=a$, $x=c, x=d$. 不可导点为 $x=b$. 在 $x=a, x=c$ 点的左右两侧, 导数 $f'(x)$ 的符号相反, 则 $x=a, x=c$ 为函数 $y=f(x)$ 的极值点. 而在 $x=b, x=d$ 点的两侧, 导数 $f'(x)$ 的符号相同, 则 $x=b, x=d$ 不是极值点, 因此, $f(x)$ 有 2 个极值点.



在 $x=b$ 点的左侧, $f'(x)$ 递减, 则 $f''(x) < 0$, 而在 $x=b$ 的右侧, $f'(x)$ 递增, 从而 $f''(x) > 0$, 故 $(b, f(b))$ 为曲线的拐点.

类似地, 在 $x=e, x=d$ 点的左、右两侧, $f'(x)$ 的单调性发生改变, 则 $f''(x)$ 的符号发生改变, 因此, $(e, f(e)), (d, f(d))$ 为曲线的拐点, 所以曲线 $y=f(x)$ 有 3 个拐点, 故应选 B.

(2) D

$$\text{解 } f'_x = \frac{e^x(x-y) - e^x}{(x-y)^2}, f'_y = \frac{0 - e^x(-1)}{(x-y)^2} = \frac{e^x}{(x-y)^2},$$

$$\text{所以 } f'_x + f'_y = \frac{e^x(x-y)}{(x-y)^2} = \frac{e^x}{x-y} = f. \quad \text{故应选 D.}$$

(3) B

解 如右图所示, 记 $D_4 = \{(x, y) \mid 0 \leq x \leq 1, \sqrt{x} \leq y \leq 1\}$,

$D_5 = \{(x, y) \mid 0 \leq x \leq 1, 0 \leq y \leq x^2\}$,

则 $D_2 = D_1 - D_4, D_3 = D_1 - D_5$.

由于 D_1 关于直线 $y=x$ 对称, 则

$$J_1 = \iint_{D_1} \sqrt[3]{x-y} dx dy = \iint_{D_1} \sqrt[3]{y-x} dx dy = -J_1, \text{ 从而 } J_1 = 0.$$

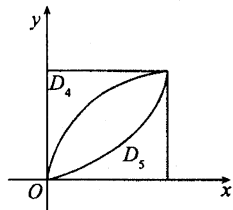
而在 D_4 上函数 $\sqrt[3]{x-y} < 0$, 从而

$$J_2 = \iint_{D_2} \sqrt[3]{x-y} dx dy = \iint_{D_1} \sqrt[3]{x-y} dx dy - \iint_{D_4} \sqrt[3]{x-y} dx dy > 0,$$

在 D_5 上函数 $\sqrt[3]{x-y} > 0$, 从而

$$J_3 = \iint_{D_3} \sqrt[3]{x-y} dx dy = \iint_{D_1} \sqrt[3]{x-y} dx dy - \iint_{D_5} \sqrt[3]{x-y} dx dy < 0,$$

因此有 $J_3 < J_1 < J_2$, 故应选 B.



(4) A

$$\text{解 由于 } \left| \left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}} \right) \sin(n+k) \right| \leq \frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}},$$

对于正项级数 $\sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}} \right)$, 其前 n 项和

$$S_n = \left(1 - \frac{1}{\sqrt{2}} \right) + \left(\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}} \right) + \cdots + \left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}} \right) = 1 - \frac{1}{\sqrt{n+1}},$$

则 $\lim_{n \rightarrow \infty} S_n = 1$, 从而级数 $\sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}} \right)$ 收敛.

由正项级数的比较判别法知级数 $\sum_{n=1}^{\infty} \left(\frac{1}{\sqrt{n}} - \frac{1}{\sqrt{n+1}} \right) \sin(n+k)$ 绝对收敛. 故应选 A.

(5) C

解 设 $P^{-1}AP = B$, 有 $(P^{-1}AP)^T = B^T$, 即有 $P^T A^T (P^T)^{-1} = B^T$, 即 A 正确;
 $(P^{-1}AP)^{-1} = B^{-1}$, 有 $P^{-1}A^{-1}P = B^{-1}$, 即 B 正确, 而 D 正确. 故应选 C.

(6) C

解 二次型的矩阵 $A = \begin{bmatrix} a & 1 & 1 \\ 1 & a & 1 \\ 1 & 1 & a \end{bmatrix}$.

由 $|\lambda E - A| = (\lambda - a - 2)(\lambda - a + 1)^2 = 0$, 得特征值为 $a + 2, a - 1$ (二重).
 由条件, $a + 2 > 0, a - 1 < 0$. 故应选 C.

(7) A

解 $P(A|B) = \frac{P(AB)}{P(B)} = 1$, 则 $P(B) = P(AB)$,

故 $P(A \cup B) = P(A) + P(B) - P(AB) = P(A)$, 所以

$$P(\bar{B}|\bar{A}) = \frac{P(\bar{B}|\bar{A})}{P(\bar{A})} = \frac{1 - P(A \cup B)}{1 - P(A)} = \frac{1 - P(A)}{1 - P(A)} = 1. \quad \text{故应选 A.}$$

(8) C

解 $D(XY) = E(X^2 Y^2) - [E(XY)]^2$
 $= E(X^2)E(Y^2) - (EX \cdot EY)^2$ (由 X 与 Y 独立)
 $= [DX + (EX)^2][DY + (EY)^2] - 1 = 14$. 故应选 C.

二、填空题

(9) 6

解 由于当 $x \rightarrow 0$ 时, $\sqrt{1 + f(x)\sin 2x} - 1 \sim \frac{1}{2}f(x)\sin 2x$, $\sin 2x \sim 2x$, $e^{3x} - 1 \sim 3x$,

$$\text{则 } \lim_{x \rightarrow 0} \frac{\sqrt{1 + f(x)\sin 2x} - 1}{e^{3x} - 1} = \lim_{x \rightarrow 0} \frac{\frac{1}{2}f(x)\sin 2x}{3x} = \lim_{x \rightarrow 0} \frac{f(x)}{3} = 2,$$

所以 $\lim_{x \rightarrow 0} f(x) = 6$.

(10) $\sin 1 - \cos 1$

解 由定积分的定义知

$$\begin{aligned} & \lim_{n \rightarrow \infty} \frac{1}{n^2} \left(\sin \frac{1}{n} + 2 \sin \frac{2}{n} + \cdots + n \sin \frac{n}{n} \right) \\ &= \lim_{n \rightarrow \infty} \frac{1}{n} \left(\frac{1}{n} \sin \frac{1}{n} + \frac{2}{n} \sin \frac{2}{n} + \cdots + \frac{n}{n} \sin \frac{n}{n} \right) \\ &= \int_0^1 x \sin x \, dx = - \int_0^1 x \, d\cos x = -x \cos x \Big|_0^1 + \int_0^1 \cos x \, dx \\ &= -\cos 1 + \sin x \Big|_0^1 \\ &= \sin 1 - \cos 1. \end{aligned}$$

(11) $-dx + 2dy$

解 等式 $(x+1)z - y^2 = x^2 f(x-z, y)$ 两边分别关于 x, y 求导,

得 $z + (x+1)z'_x = 2xf(x-z, y) + x^2 f'_1(x-z, y) \cdot (1-z'_z)$;

$(x+1)z'_y - 2y = x^2[f'_1(x-z, y) \cdot (-z'_y) + f'_2(x-z, y)]$.

再将 $x=0, y=1$ 代入原式, 可得 $z=1$.

将 $x=0, y=1, z=1$ 代入上述两式, 得 $z'_x=-1, z'_y=2$.

故 $dz|_{(0,1)} = z'_x dx + z'_y dy = -dx + 2dy$.

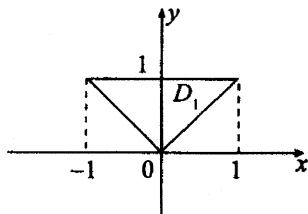
(12) $\frac{1}{3} - \frac{2}{3e}$

解 积分区域 D 如右图所示, D 关于 y 轴对称, 记

$D_1 = \{(x, y) \mid 0 \leq x \leq 1, x \leq y \leq 1\}$,

由对称性知

$$\begin{aligned} \iint_D x^2 e^{-y^2} dx dy &= 2 \iint_{D_1} x^2 e^{-y^2} dx dy = 2 \int_0^1 dy \int_0^y x^2 e^{-y^2} dx \\ &= \frac{2}{3} \int_0^1 y^3 e^{-y^2} dy \stackrel{\text{令 } y^2=t}{=} \frac{1}{3} \int_0^1 t e^{-t} dt = -\frac{1}{3} \int_0^1 t de^{-t} \\ &= -\frac{1}{3} t e^{-t} \Big|_0^1 + \frac{1}{3} \int_0^1 e^{-t} dt = \frac{1}{3} - \frac{2}{3e}. \end{aligned}$$



(13) $\lambda^4 + \lambda^3 + 2\lambda^2 + 3\lambda + 4$

解 按最后一行展开, 得

$$\begin{aligned} &(-1)^{4+1} \times 4 \begin{vmatrix} -1 & 0 & 0 \\ \lambda & -1 & 0 \\ 0 & \lambda & -1 \end{vmatrix} + (-1)^{4+2} \times 3 \begin{vmatrix} \lambda & 0 & 0 \\ 0 & -1 & 0 \\ 0 & \lambda & -1 \end{vmatrix} + \\ &(-1)^{4+3} \times 2 \begin{vmatrix} \lambda & -1 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & -1 \end{vmatrix} + (-1)^{4+4} (\lambda+1) \begin{vmatrix} \lambda & -1 & 0 \\ 0 & \lambda & -1 \\ 0 & 0 & \lambda \end{vmatrix} \\ &= \lambda^4 + \lambda^3 + 2\lambda^2 + 3\lambda + 4. \end{aligned}$$

(14) $\frac{2}{9}$

解 取球次数恰好为 4 次, 说明第 4 次有一种颜色, 前 3 次有两种颜色.

由古典概率公式, 得 $P = \frac{C_3^1 \cdot C_3^1 \cdot 2}{3^4} = \frac{2}{9}$.

三、解答题

(15) 解 因为 $\lim_{x \rightarrow 0} (\cos 2x + 2x \sin x)^{\frac{1}{x^4}} = e^{\lim_{x \rightarrow 0} \frac{\ln(\cos 2x + 2x \sin x)}{x^4}}$

$$\begin{aligned} \text{且 } \lim_{x \rightarrow 0} \frac{\ln(\cos 2x + 2x \sin x)}{x^4} &= \lim_{x \rightarrow 0} \frac{1}{\cos 2x + 2x \sin x} \cdot \frac{-2\sin 2x + 2\sin x + 2x \cos x}{4x^3} \\ &= \lim_{x \rightarrow 0} \frac{-2\cos 2x + 2\cos x - x \sin x}{6x^2} \\ &= \lim_{x \rightarrow 0} \frac{4\sin 2x - 3\sin x - x \cos x}{12x} \\ &= \frac{1}{3}, \end{aligned}$$

所以 $\lim_{x \rightarrow 0} (\cos 2x + 2x \sin x)^{\frac{1}{x^4}} = e^{\frac{1}{3}}$.

(16) 解 (I) 由题设 $-\frac{p}{Q} \cdot \frac{dQ}{dp} = \frac{p}{120-p}$.

所以 $\int \frac{dQ}{Q} = -\int \frac{1}{120-p} dp$, 可得 $\ln Q = \ln(120-p) + \ln C$, 即 $Q = C(120-p)$.

又最大需求量为 1200, 故 $C=10$, 所以需求函数 $Q=1200-10p$.

(II) 由(I)知, 收益函数 $R=120Q-\frac{1}{10}Q^2$, 边际收益 $R'(Q)=120-\frac{1}{5}Q$.

当 $p=100$ 时, $Q=200$, 故当 $p=100$ 万元时的边际收益 $R'(200)=80$, 其经济意义为: 销售第 201 件商品所得的收益为 80 万元.

(17) 解 当 $0 < x \leq 1$ 时,

$$f(x) = \int_0^x |t^2 - x^2| dt + \int_x^1 |t^2 - x^2| dt = \int_0^x (x^2 - t^2) dt + \int_x^1 (t^2 - x^2) dt = \frac{4}{3}x^3 - x^2 + \frac{1}{3};$$

$$\text{当 } x > 1 \text{ 时, } f(x) = \int_0^1 (x^2 - t^2) dt = x^2 - \frac{1}{3},$$

$$\text{所以 } f(x) = \begin{cases} \frac{4}{3}x^3 - x^2 + \frac{1}{3}, & 0 < x \leq 1, \\ x^2 - \frac{1}{3}, & x > 1, \end{cases}$$

$$\text{而 } f'_-(1) = \lim_{x \rightarrow 1^-} \frac{\frac{4}{3}x^3 - x^2 + \frac{1}{3} - \frac{2}{3}}{x-1} = 2, f'_+(1) = \lim_{x \rightarrow 1^+} \frac{x^2 - \frac{1}{3} - \frac{2}{3}}{x-1} = 2,$$

$$\text{故 } f'(x) = \begin{cases} 4x^2 - 2x, & 0 < x \leq 1, \\ 2x, & x > 1, \end{cases}$$

由 $f'(x)=0$ 求得唯一驻点 $x=\frac{1}{2}$, 又 $f''(\frac{1}{2}) > 0$, 从而 $x=\frac{1}{2}$ 为 $f(x)$ 的最小值点, 最小

值为 $f(\frac{1}{2}) = \frac{1}{4}$.

(18) 解 令 $u=x-t$, 则 $\int_0^x f(x-t) dt = \int_0^x f(u) du$.

$$\text{由题设 } \int_0^x f(u) du = x \int_0^x f(t) dt - \int_0^x t f(t) dt + e^{-x} - 1,$$

$$\text{求导得 } f(x) = \int_0^x f(t) dt - e^{-x}, \text{ 且 } f(0) = -1.$$

$$\text{因此 } f'(x) - f(x) = e^{-x},$$

$$\begin{aligned} \text{从而 } f(x) &= e^{\int dx} \left(C + \int e^{-x} e^{-\int dx} dx \right) \\ &= Ce^x - \frac{e^{-x}}{2}. \end{aligned}$$

$$\text{由 } f(0) = -1, \text{ 得 } C = -\frac{1}{2}, \text{ 所以 } f(x) = -\frac{1}{2}(e^x + e^{-x}).$$

(19) 解 因为 $\lim_{n \rightarrow \infty} \left| \frac{\frac{x^{2n+4}}{(n+2)(2n+3)}}{\frac{x^{2n+2}}{(n+1)(2n+1)}} \right| = x^2$, 所以当 $|x| < 1$ 时, 幂级数绝对收敛; 当 $|x| > 1$ 时,

幂级数发散.

又当 $x = \pm 1$ 时, 级数 $\sum_{n=0}^{\infty} \frac{1}{(n+1)(2n+1)}$ 收敛, 所以幂级数的收敛域为 $[-1, 1]$.

记 $f(x) = \sum_{n=0}^{\infty} \frac{x^{2n+2}}{(n+1)(2n+1)}, x \in [-1, 1]$, 则

$$f'(x) = 2 \sum_{n=0}^{\infty} \frac{x^{2n+1}}{2n+1}, f''(x) = 2 \sum_{n=0}^{\infty} x^{2n} = \frac{2}{1-x^2}, x \in (-1, 1).$$

因为 $f'(0) = 0, f(0) = 0$, 所以当 $x \in (-1, 1)$ 时,

$$f'(x) = \int_0^x f''(t) dt = \int_0^x \frac{2}{1-t^2} dt = \ln(1+x) - \ln(1-x),$$

$$f(x) = \int_0^x f'(t) dt = \int_0^x \ln(1+t) dt - \int_0^x \ln(1-t) dt = (1+x)\ln(1+x) + (1-x)\ln(1-x).$$

$$\text{又 } f(1) = \lim_{x \rightarrow 1^-} f(x) = 2\ln 2, f(-1) = \lim_{x \rightarrow -1^+} f(x) = 2\ln 2,$$

$$\text{所以 } f(x) = \begin{cases} (1+x)\ln(1+x) + (1-x)\ln(1-x), & x \in (-1, 1), \\ 2\ln 2, & x = \pm 1. \end{cases}$$

(20) 解 (I) 对矩阵 $(A \mid \beta)$ 施以初等行变换

$$(A \mid \beta) = \left(\begin{array}{ccc|c} 1 & 1 & 1-a & 0 \\ 1 & 0 & a & 1 \\ a+1 & 1 & a+1 & 2a-2 \end{array} \right) \rightarrow \left(\begin{array}{ccc|c} 1 & 1 & 1-a & 0 \\ 0 & -1 & 2a-1 & 1 \\ 0 & 0 & -a^2+2a & a-2 \end{array} \right),$$

由方程组无解知, 秩 $(A \mid \beta) > \text{秩 } A$, 即 $-a^2 + 2a = 0$, 且 $a-2 \neq 0$, 解得 $a = 0$.

(II) 对矩阵 $(A^T A \mid A^T \beta)$ 施以初等行变换

$$(A^T A \mid A^T \beta) = \left(\begin{array}{ccc|c} 3 & 2 & 2 & -1 \\ 2 & 2 & 2 & -2 \\ 2 & 2 & 2 & -2 \end{array} \right) \rightarrow \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & -2 \\ 0 & 0 & 0 & 0 \end{array} \right),$$

$$\text{所以, 方程组 } A^T A x = A^T \beta \text{ 的通解 } x = \begin{pmatrix} 1 \\ -2 \\ 0 \end{pmatrix} + k \begin{pmatrix} 0 \\ -1 \\ 1 \end{pmatrix} \quad (k \text{ 为任意常数}).$$

(21) 解 (I) 因为

$$|\lambda E - A| = \begin{vmatrix} \lambda & 1 & -1 \\ -2 & \lambda+3 & 0 \\ 0 & 0 & \lambda \end{vmatrix} = \lambda(\lambda+1)(\lambda+2),$$

所以 A 的特征值为 $\lambda_1 = -1, \lambda_2 = -2, \lambda_3 = 0$.

当 $\lambda_1 = -1$ 时, 解方程组 $(-E - A)x = 0$, 得特征向量 $\xi_1 = (1, 1, 0)^T$;

当 $\lambda_2 = -2$ 时, 解方程组 $(-2E - A)x = 0$, 得特征向量 $\xi_2 = (1, 2, 0)^T$;

当 $\lambda_3 = 0$ 时, 解方程组 $Ax = 0$, 得特征向量 $\xi_3 = (3, 2, 2)^T$.

$$\text{令 } P = (\xi_1, \xi_2, \xi_3) = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 2 & 2 \\ 0 & 0 & 2 \end{pmatrix}, \text{ 则}$$

$$P^{-1}AP = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\begin{aligned}
\text{所以 } \mathbf{A}^{99} &= \mathbf{P} \begin{pmatrix} (-1)^{99} & 0 & 0 \\ 0 & (-2)^{99} & 0 \\ 0 & 0 & 0 \end{pmatrix} \mathbf{P}^{-1} \\
&= \begin{pmatrix} 1 & 1 & 3 \\ 1 & 2 & 2 \\ 0 & 0 & 2 \end{pmatrix} \begin{pmatrix} (-1)^{99} & 0 & 0 \\ 0 & (-2)^{99} & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 2 & -1 & -2 \\ -1 & 1 & \frac{1}{2} \\ 0 & 0 & \frac{1}{2} \end{pmatrix} \\
&= \begin{pmatrix} 2^{99} - 2 & 1 - 2^{99} & 2 - 2^{98} \\ 2^{100} - 2 & 1 - 2^{100} & 2 - 2^{99} \\ 0 & 0 & 0 \end{pmatrix}
\end{aligned}$$

(II) 因为 $\mathbf{B}^2 = \mathbf{B}\mathbf{A}$, 所以

$$\mathbf{B}^{100} = \mathbf{B}^{98} \mathbf{B}^2 = \mathbf{B}^{99} \mathbf{A} = \mathbf{B}^{97} \mathbf{B}^2 \mathbf{A} = \mathbf{B}^{98} \mathbf{A}^2 = \cdots = \mathbf{B} \mathbf{A}^{99},$$

$$\text{即 } (\boldsymbol{\beta}_1, \boldsymbol{\beta}_2, \boldsymbol{\beta}_3) = (\boldsymbol{\alpha}_1, \boldsymbol{\alpha}_2, \boldsymbol{\alpha}_3) \begin{pmatrix} 2^{99} - 2 & 1 - 2^{99} & 2 - 2^{98} \\ 2^{100} - 2 & 1 - 2^{100} & 2 - 2^{99} \\ 0 & 0 & 0 \end{pmatrix}$$

所以

$$\begin{cases} \boldsymbol{\beta}_1 = (2^{99} - 2)\boldsymbol{\alpha}_1 + (2^{100} - 2)\boldsymbol{\alpha}_2, \\ \boldsymbol{\beta}_2 = (1 - 2^{99})\boldsymbol{\alpha}_1 + (1 - 2^{100})\boldsymbol{\alpha}_2, \\ \boldsymbol{\beta}_3 = (2 - 2^{98})\boldsymbol{\alpha}_1 + (2 - 2^{99})\boldsymbol{\alpha}_2, \end{cases}$$

(22) 解 (I) (X, Y) 的概率密度为

$$f(x, y) = \begin{cases} 3, & (x, y) \in D, \\ 0, & \text{其他}. \end{cases}$$

(II) 对于 $0 < t < 1$,

$$\begin{aligned}
P\{U \leq 0, X \leq t\} &= P\{X > Y, X \leq t\} \\
&= \int_0^t dx \int_{x^2}^x 3 dy \\
&= \frac{3}{2} t^2 - t^3,
\end{aligned}$$

$$P\{U \leq 0\} = P\{X > Y\} = \frac{1}{2},$$

$$P\{X \leq t\} = \int_0^t dx \int_{x^2}^{\sqrt{x}} 3 dy = 2t^{\frac{3}{2}} - t^3.$$

由于 $P\{U \leq 0, X \leq t\} \neq P\{U \leq 0\}P\{X \leq t\}$, 所以 U 与 X 不相互独立.

(III) 当 $z < 0$ 时, $F(z) = 0$; 当 $0 \leq z < 1$ 时,

$$\begin{aligned}
F(z) &= P\{Z \leq z\} \\
&= P\{U + X \leq z\} \\
&= P\{U = 0, X \leq z\} \\
&= P\{X > Y, X \leq z\} \\
&= \frac{3}{2} z^2 - z^3;
\end{aligned}$$

当 $1 \leq z < 2$ 时, $F(z) = P\{U + X \leq z\}$

$$\begin{aligned}
 &= P\{U=0, X \leq z\} + P\{U=1, X \leq z-1\} \\
 &= \frac{1}{2} + 2(z-1)^{\frac{3}{2}} - \frac{3}{2}(z-1)^2;
 \end{aligned}$$

当 $z \geq 2$ 时, $F(z) = P\{U+X \leq z\} = 1$.

$$\text{所以 } F(z) = \begin{cases} 0, & z < 0, \\ \frac{3}{2}z^2 - z^3, & 0 \leq z < 1, \\ \frac{1}{2} + 2(z-1)^{\frac{3}{2}} - \frac{3}{2}(z-1)^2, & 1 \leq z < 2, \\ 1, & z \geq 2. \end{cases}$$

(23) 解 (I) 总体 X 的分布函数为

$$F(x) = \begin{cases} 0, & x < 0, \\ \frac{x^3}{\theta^3}, & 0 \leq x < \theta, \\ 1, & x \geq \theta. \end{cases}$$

从而 T 的分布函数为

$$F_T(z) = [F(z)]^3 = \begin{cases} 0, & z < 0, \\ \frac{z^9}{\theta^9}, & 0 \leq z < \theta, \\ 1, & z \geq \theta. \end{cases}$$

所以 T 的概率密度为

$$f_T(z) = \begin{cases} \frac{9z^8}{\theta^9}, & 0 < z < \theta, \\ 0, & \text{其他}. \end{cases}$$

$$\begin{aligned}
 \text{(II) } E(T) &= \int_{-\infty}^{+\infty} z f_T(z) dz \\
 &= \int_0^\theta \frac{9z^9}{\theta^9} dz \\
 &= \frac{9}{10} \theta,
 \end{aligned}$$

从而 $E(aT) = \frac{9}{10} a \theta$.

令 $E(aT) = \theta$, 得 $a = \frac{10}{9}$.

所以当 $a = \frac{10}{9}$ 时, $E(aT) = \theta$.